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The reflexive model of social behavior under informational influence

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Abstract

This paper proposes a mathematical model of the reflexive interaction of some society groups that can act as passive or active agents of influence. The behavior of a set of active agents aimed at imposing a certain behavior on other groups of society that differ in their response to information influence is studied. It was taken into account that passive agents turn into active ones under the influence. In analyzing the information influence of active agents on passive ones, the internal currency was chosen as the main factor. As a result of the numerical experiment is the changes in the model over time are graphically shown, in particular, discrete and general influence views and efficiency of a given information influence are obtained.

Keywords: reflexive game, internal currency, information influence, agents of influence.

Introduction

In modern realities, the impact on the behavior and thoughts of different strata of society has significantly intensified through air alarms, the availability of shelters, the situation with the operability of the energy system, etc. In such situations, fear as an instinct of self-preservation becomes one of the main factors for decision-making, and therefore awareness becomes an important factor that influences the actions and deeds of people. Trying to find an answer to certain questions about their safety, in addition to their internal perception, they are often guided by the position of their environment. Other people, their words and actions can influence our choice, consciously or not, even sometimes to the point of complete refusal to choose.

In this case, the indicated situation can be considered from the point of view of a reflexive game, which was described in the work [1]. In such a game, agents (players) interact with each other, knowing the measure of the significance of their values and taking into account their perception of each other. Such perception is described in this work as the first level of reflection. For the numerical representation of the information influence, the internal currency is used.

Based on the models of hidden and informational control, considered in [2, 3], it is proposed to describe the impact in reflexive games. In this work, various types of agents are considered, including active agents that control the subject in the process of informational influence, and passive agents that are the object of control. In addition, the transition from a passive to an active agent under the influence of informational influence is analyzed.

Theoretical part

1.1. Internal currency

Internal currency is a numerical characteristic of the values of agents that is a regulator of the activities of other agents. The problem of calculating the internal currency is not well covered in the scientific literature. This is due to the fact that for a long time this approach to calculating the values of agents has not been used. Hence, it is of interest to try to apply it to the calculation of certain parameters of a reflexive model of social behavior. The mathematical model, first proposed by Lefebvre [1], looks as follows. Let players X and Y receive nominal gains A and B, respectively. The values α and β are introduced, where the parameter α characterizes the relationship of player X to himself, and the parameter β - his relationship to the opponent. The same parameters are introduced for player Y. The internal currency of each of them is built according to the following rule:

$$H_1^{(X)} = A + \alpha A + \beta B,$$
$$H_1^{(Y)} = B + \alpha B + \beta A,$$

where $H_1^{(X)}$ and $H_1^{(Y)}$, respectively, are the internal currencies of players X and Y, point of view of player .

Let us describe the sets of agents. Let us assume that there are three groups of agents - θ_1, θ_2, θ_3:

- θ_1 is the set of active agents;
- θ_2 is the set of passive agents;
- θ_3 is the set of passive agents that have a threshold of limited influence.

The total number of agents is $N = N_1 + N_2 + N_3$, where N_i is the number of agents in set θ_i, $i = \overline{1,3}$. Taking into account the certain conditionality of the model built by Lefebvre, we will consider the influence of agents θ_1 on θ_2 and θ_3 from the point of view of internal currency, taking as a basis the model of internal currency taking into account the first-rank reflection, proposed in [4, 5]:

$H^{\theta_i} = \sum_j \alpha_{ij} A_{ij} + \sum_k \beta_{ik} B_{ik}, = 2, 3$, where:

- H^{θ_i} is the internal currency of the i-th group of agents;
- A_{ij} is the j-th nominal gain of the i-th group of agents;
- α_{ij} is the significance measure of A_{ij} for θ_i;
- B_{ik} is the k-th nominal gain of other agents from the point of view of θ_i;
- $\beta_{ik}$is the significance measure of B_{ik} from the point of view of θ_i.

Therefore, analyzing the structure of internal currency, we see that the first addend $\sum_j \alpha_{ij} A_{ij}$is the own vision of the situation, the confidence in one's own opinion, which depends on the internal representations of a person, and the second addend $\sum_k \beta_{ik} B_{ik}$ is a reflection of the external influence on the person's vision of the situation, narratives imposed on her. Thus, having an understanding of the elements of which the internal currency consists, active agents can influence passive ones. This work is dedicated to the study of this influence. Taking into account the previous investigation [6], we will assume that the measures of significance α_{ij} and β_{ik} depend on the number of representatives of the sets θ_2 and θ_3 who were exposed to influence at time t, that is: $\alpha_{ij} = \alpha_{ij}(x_t)$ i $\beta_{ik} = \beta_{ik}(x_t)$.

Let the following equations hold:

$$\sum_k \beta_{ik} B_{ik} = b_i \beta_i(x_t) \#(1)$$
$$\sum_j \alpha_{ij} A_{ij} = a_i \alpha_i(x_t), \#(2)$$

where a_i, $b_i =$ const.

Then, the expression for the internal currency will be rewritten as follows:

$$H^{\theta_i} = a_i \alpha_i(x_t) + b_i \beta_i(x_t) \#(3)$$

1.2. Internal currency thresholds

Based on formula (3), we calculate the maximum values of the internal currency for the groups θ_2 and θ_3 which we denote by $H_{max}^{\theta_2}$ and $H_{max}^{\theta_3}$ respectively. Knowing these values, we introduce the thresholds of the internal currency $h_i^{\theta_i} = \left\{h_1^{\theta_i}, h_2^{\theta_i}, \dots, h_l^{\theta_i}\right\}$, $i = 2, 3$. Taking into account the work [7], the following thresholds were set for the impact on members of society:

$$h^{\theta_2} = \left\{0.3 \cdot H_{max}^{\theta_2}, 0.55 \cdot H_{max}^{\theta_2}\right\},$$
$$h^{\theta_3} = \left\{0.3 \cdot H_{max}^{\theta_3}, 0.45 \cdot H_{max}^{\theta_3}\right\}.$$

1.3. Determining the significance measure for nominal gains within groups of agents

Influence on other groups is determined by the fact that the enemy tries to impose its narratives by replacing values. In this case, to achieve its goals, it needs to reduce the value of (2) and increase the value of (1).

Therefore, as $\alpha_i(x_t)$ the following is proposed:

$$\alpha_2(x_t) = \begin{cases} cos\frac{\pi x_t}{2N_2}, x_t < N_2 \\ 0, x_t \geq N_2 \end{cases} \#(4)$$

$$\alpha_3(x_t) = \begin{cases} cos\frac{\pi x_t}{2N_3}, x_t < N_3 \\ 0.45 \cdot cos^2\left(\frac{1}{2} arctg\frac{2b_3}{a_3}\right), x_t \geq N_3 \end{cases} \#(5)$$

It should be noted that in formulas (4) and (5) under the conditions $x_t \geq N_2$ and $x_t \geq N_3$ the values of the functions selected from the behavior of the groups take on the following values: 0 in formula (4) due to the actual decline in the values of the representatives of the set θ_2 and $cos^2(\frac{1}{2} arctg \frac{2b_3}{a_3})$ in (5), which is not equal to zero, as in formula (4), since when this threshold value is reached, the representatives of the set θ_3 do not return to the game.

As $\beta_i(x_t)$ the following is proposed:

$$\beta_2(x_t) = \begin{cases} sin \frac{\pi x_t}{N_2}, x_t < N_2 \\ 0, x_t \geq N_2 \end{cases} \#(6)$$

$$\beta_3(x_t) = \begin{cases} sin \frac{\pi x_t}{N_3}, x_t < N_3 \\ 0.45 \cdot sin\left(\frac{1}{2} arctg \frac{2b_3}{a_3}\right), x_t \geq N_3 \end{cases} \#(7)$$

Similarly to formulas (4) and (5) under the conditions $x_t \geq N_2$ and $x_t \geq N_3$ in expressions (6) and (7), the values $\beta_i(x_t)$ take on the values 0 i $0.45 \cdot sin(\frac{1}{2} arctg \frac{2b_3}{a_3})$.

It should also be noted that after passing the first threshold, the measure of the importance of self-confidence is further influenced by changing the dependence that transforms expressions (4) and (5), namely:

$$\alpha_2(x_t) = \begin{cases} cos^2 \frac{\pi x_t}{2N_2}, x_t < N_2 \\ 0, x_t \geq N_2 \end{cases} \#(8)$$

$$\alpha_3(x_t) = \begin{cases} cos^2 \frac{\pi x_t}{2N_3}, x_t < N_3 \\ 0.45 cos^2\left(\frac{1}{2} arctg \frac{2b_3}{a_3}\right), x_t \geq N_3 \end{cases} \#(9)$$

This will change the values of $H_{max}^{\theta_2}$ й $H_{max}^{\theta_3}$ downward, which will further strengthen the influence exerted by the agents of the group θ_1.

The differences in formulas (4) – (9) for different groups are related to their behavioral patterns. Thus, the group of agents θ_2 is more susceptible to information influence, so its internal currency is significantly adjusted by the actions of the enemy, especially after passing the thresholds. At the same time, a group of agents θ_3 due to the existing threshold of limited external pressure, after passing the second threshold of its internal currency, it ceases to respond to the influence, and as a result, its internal currency becomes stable.

1.4. Recalculating the number of agents

Let us denote by $x^1(t)$ the function that determines the number of agents of influence at a given time t. At different moments of time, groups from the set θ_1.

$$x^1(t) = x_t^1(t_0, n_1, n_2) = \begin{cases} n_1, t \leq t_0 \\ n_2, t > t_0 \end{cases}$$

where:

- t_0 is the projected time of the beginning of the peak impact;
- n_1 is the number of members of the influence group before the peak of influence;
- n_2 is the number of members of the influence group during the peak of influence.

The sizes of the influence groups are selected as follows:

1. n_1 - large enough to reach the peak of influence as soon as possible;
2. n_2 – more that n_1 and is approaching the ideal value;
3. ideal value $n_2 = \frac{3 \cdot N_1 - n_1 \cdot t_0}{\Delta t_0}$, Δt_0- is the duration of the exposure peak.

In addition, we believe that agents with θ_1 have the ability to exert influence three times, so for the convenience of calculations, we add two dummy agents to each θ_1 we add two fictitious ones to each agent. Thus, the actual number of elements in the set θ_1 in terms of θ_2 i θ_3 increases from N_1 to $3N_1$.

Let us denote x_t^2 i x_t^3 functions that determine the number of representatives, respectively, of the sets θ_2 i $\theta_3$that were exposed at the moment of time t.

As $x_t^2(x^1(t))$ we will consider:

$$x_t^2 = \begin{cases} \left[\frac{x^1(t)}{5}\right], \mathrm{H}^{\theta_2} < 0.3H_{max}^{\theta_2} \\ 2\left[\frac{x^1(t)}{3}\right], 0.3H_{max}^{\theta_2} \leq \mathrm{H}^{\theta_2} < 0.55H_{max}^{\theta_2} \\ 3x^1(t),\ 0.55H_{max}^{\theta_2} \leq \mathrm{H}^{\theta_2} \end{cases}$$

As $x_t^3(x^1(t))$ we will consider:

$$x_t^3 = \begin{cases} \left[\frac{x^1(t)}{5}\right], \mathrm{H}^{\theta_3} < 0.3H_{max}^{\theta_3} \\ 2\left[\frac{x^1(t)}{3}\right], 0.3H_{max}^{\theta_3} \leq \mathrm{H}^{\theta_3} < 0.45H_{max}^{\theta_3} \\ 0,\ 0.45H_{max}^{\theta_3} \leq \mathrm{H}^{\theta_3} \end{cases}$$

In the above formulas, the first expression $\left[\frac{x^1(t)}{5}\right]$ means that before the first threshold value, five agents from the group θ_1 completely take under their influence one representative of the group $\theta_i$thus reducing the corresponding set by one element. Between the first and second thresholds, the influence increases, respectively, to the value $2\left[\frac{x^1(t)}{3}\right]$. The situation changes after the second threshold is passed. The set θ_2 loses $3x^1(t)$ members, while the set θ_3, on the contrary, does not lose any members, i.e., we assume that after a certain value of the internal currency, agents from θ_3 are not influenced by the oversaturation of agitation, propaganda, etc.

Let us now consider how changes occur in the sets θ_i, $i = \underline{1,3}$. Let us denote by $k_i(t)$ the current number of agents in the group θ_i. Then we have:

$$k_1(t+1) = k_1(t) - x^1(t), k_1(0) = 3N_1.$$

The calculation of $k_2(t)$ i $k_3(t)$ is calculated in two stages, since the agents from $\theta_1$and then those that are affected, that is, additionally affect together each of the sets θ_2 i θ_3. Hence:

- the first stage: the values of $k_2(t)$ i $k_3(t)$ decrease due to the influence of $x^1(t)$.
- the second stage: each of the values $k_2(t)$ i $k_3(t)$ is further reduced due to the influence of $x_t^2(x^1(t))$ i $x_t^3(x^1(t))$.

Limiting ourselves to only one stage of secondary impact, we obtain the formulas for recalculating $k_2(t)$ i $k_3(t)$ under the initial conditions $k_2(0) = N_2$, $k_3(0) = N_3$:

$$k_2(t+1) = k_2(t) - x_t^2(x^1(t)) - x_t^2\left(x_t^2(x^1(t)) + x_t^3(x^1(t))\right),$$

$$k_3(t+1) = k_3(t) - x_t^3(x^1(t)) - x_t^3\left(x_t^2(x^1(t)) + x_t^3(x^1(t))\right).$$

We express by $x(t)$ the current number of agents interacting at a given time t:

$$x(t) = x^1(t) + x_t^2(x^1(t)) + \\ + x_t^2\left(x_t^2(x^1(t)) + x_t^3(x^1(t))\right) + \\ + x_t^3(x^1(t)) + x_t^3\left(x_t^2(x^1(t)) + x_t^3(x^1(t))\right).$$

Practical part

2.1. Model parameters

Let us consider the application of the developed model in a numerical experiment.

The number of agents: $N = 10000$, $N_1 = 0.05 \cdot N, N_2 = 0.8 \cdot N, N_3 = 0.15 \cdot N$. Influence agents act in groups according to the following parameters: $n_1 = 20, n_2 = 125, t_0 = 25, \Delta t_0 = 8$. Parameters of internal currencies: a_2 = 20, a_3 = 25, b_2 = 75, b_3 = 90.

The effectiveness of a given influence will be calculated as the proportion of the number of agents that were affected and their initial total number. The results are shown in the form of graphs.

2.2. Modeling results

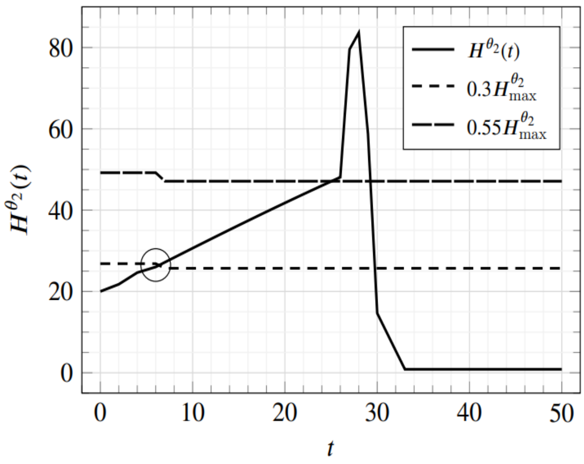

Figure 1: Internal currency of group θ_2

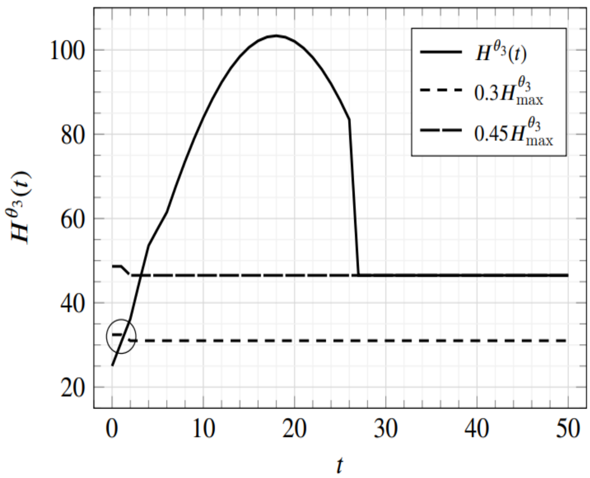

Figure 2: Internal currency of group θ_3

Figure 1 and **Figure 2**: Internal currency of group θ_3Figure 2 show how the internal currency of passive agents changes. The first line is the respective internal currency, the second line is the first threshold, and the third line is the second threshold. The behavior of the groups under influence is fully regulated by the values of their internal currencies, as will be demonstrated below. It should be noted that the circle marks the moment when, upon passing the first threshold, a change in $\alpha_i(x_t)$ from $cos\frac{\pi x_t}{2N_i}$ to $cos^2\frac{\pi x_t}{2N_i}$.

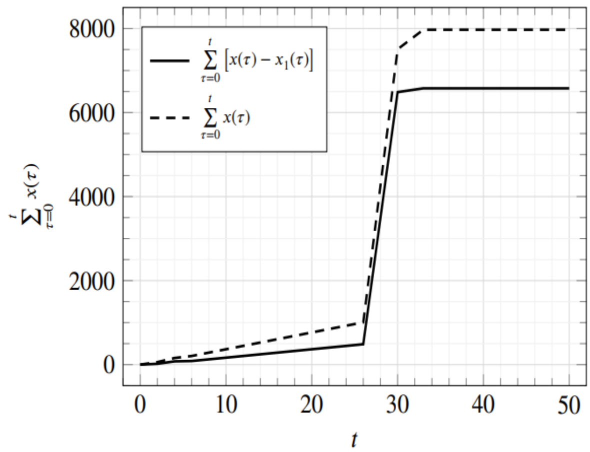

Figure 3: General informational influence view

In Figure 3, the solid curve is the total number of agents that have been influenced over all iterations, and the dashed curve is the number of agents together with the influencers. From here, it is easy to see the process of increasing the number of agents that have been influenced, as well as the peak of influence.

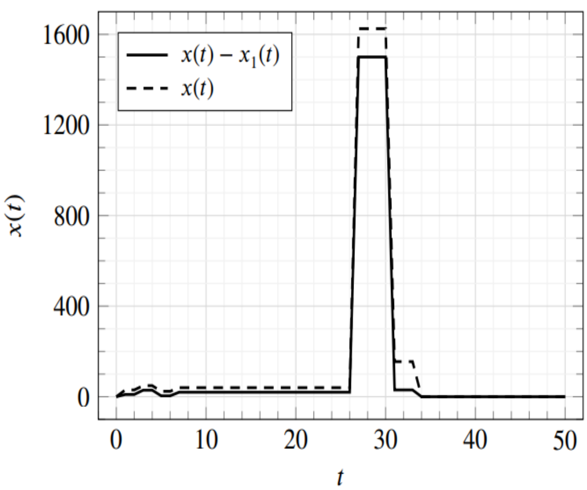

Figure 4: Discrete informational influence view

In Figure 4, the solid curve is the number of agents that were affected at the moment t, dotted curve - the number of agents that were exposed together with the agents of influence at the moment t. The work of influence agents can be seen as the difference between the values of the dashed and solid curves.

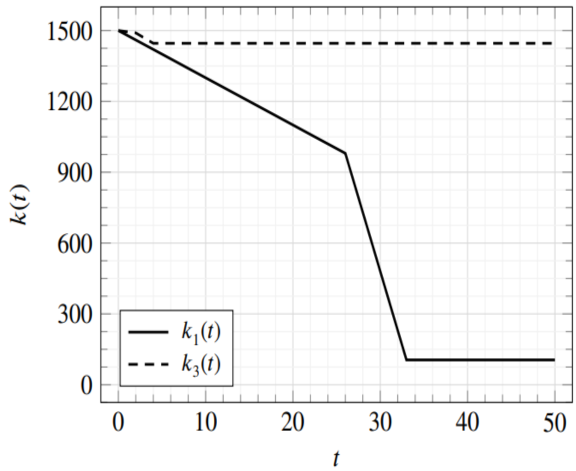

Figure 5: Change in the current number of agents in θ_1 and θ_3 groups

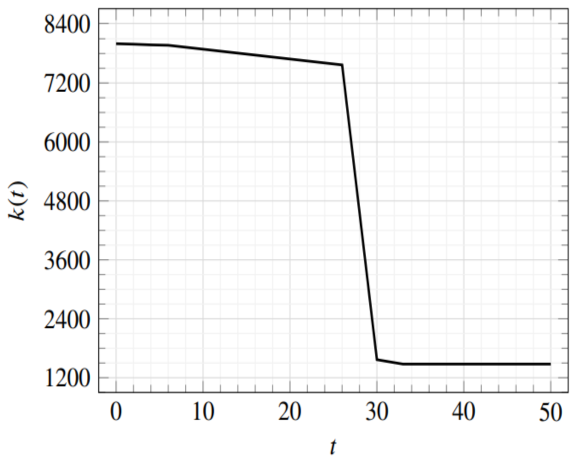

Figure 6: Change in the current number of agents in θ_2 group

Figure 5 and Figure 6 show the behavior of different groups of agents, which is conditioned by changes in internal currencies (Figure 1 and **Figure 2**: Internal currency of group θ_3Figure 2). In Figure 5, the solid curve is the behavior of the agents of influence, who start cautiously, waiting for a strong reaction from the second group until its internal currency crosses the second threshold, and then increase their influence. At the same time, in Figure 6, the second group fully responds to the influence exerted on them, the greater the influence, the greater the reaction. The third group, the dotted curve in Figure 5, participates in the game only for the first few iterations until its internal currency crosses the second threshold. Its reaction to the massive impact is to leave the game.

Conclusions

A mathematical model of the information influence of active agents on passive ones at the expense of their internal currencies is developed. A numerical experiment and its parameters are presented, which resulted in an information influence with an efficiency of 69.22%, where the effectiveness of a given impact is understood as a value defined as the proportion of the number of agents exposed to the impact to their initial total number. The impact on the groups of agents θ_2 i θ_3 with different reactions to the massive impact on the part of the agents of the group θ_1. The study of this model shows the possibility of its practical application. Such a model can be used to predict the information impact and the ways of its application on certain segments of the population.

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