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Price Impact for Different Market Models in Cryptocurrency Trading

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Abstract

Price impact in cryptocurrency trading plays a crucial role in understanding market dynamics and liquidity. The study presents a detailed mathematical analysis of price impact across three different market models: constant sum, constant product, and order book. Each model provides a unique perspective on how asset prices are influenced by trade execution, market depth, and available liquidity. By examining these models, the analysis highlights the relationship between trade volume and price changes, offering important insights into how large transactions affect the stability and behavior of prices in various liquidity environments. The results are relevant for traders and investors aiming to optimize their strategies in volatile markets, as well as for regulators seeking to mitigate the systemic risks posed by large-scale trades in the cryptocurrency space.

Keywords: price impact, constant product, constant sum, order book.

Introduction

Price impact is a fundamental concept in financial markets, representing the change in the price of an asset due to the execution of a trade or a series of trades. Unlike simple price movements caused by market sentiment or external news, price impact specifically refers to the direct influence that a trade has on an asset's price. This concept is especially relevant in markets that exhibit high volatility or have varying levels of liquidity, such as the cryptocurrency market.

The study of price impact is essential for several reasons. Firstly, it allows traders, investors, and market makers to understand how their trades might influence market prices, which is crucial for executing large orders without causing excessive price fluctuations. Secondly, understanding price impact helps in assessing the liquidity of an asset—assets with high price impact are typically less liquid, meaning that buying or selling large quantities can significantly alter their prices. Thirdly, price impact analysis is a valuable tool for risk management, enabling traders to anticipate and mitigate potential losses arising from their trades. Lastly, regulators can use price impact analysis to monitor market health and detect manipulative activities, ensuring the integrity of financial markets.

Price impact has been the subject of extensive research across various financial markets. In traditional equity markets, seminal works such as those by Kyle (1985) [1] introduced models to quantify the impact of trades on prices, laying the foundation for understanding market microstructure. Kyle's model, often referred to as the "Kyle Model," suggests that larger trades have a greater impact on prices, particularly in less liquid markets. This model has been widely used to study how information asymmetry and liquidity affect price impact.

In the context of cryptocurrency markets, research has been more recent but rapidly growing. Studies by Schmitt et al. (2019) [7] and Wang and Zhang (2020) [9] have explored how the unique characteristics of cryptocurrency markets—such as high volatility, fragmented liquidity, and 24/7 trading—affect price impact. These studies have shown that the price impact in cryptocurrency markets can be significantly higher than in traditional financial markets, primarily due to lower liquidity and higher volatility. For instance, Schmitt et al. [7] found that the price impact of trades in Bitcoin is more pronounced during periods of low liquidity, which can lead to significant price swings and increased market instability.

Moreover, the evolving nature of cryptocurrency markets has led to the development of new models and approaches to studying price impact. Recent work by Donier and Bouchaud (2015) [6] introduced models that account for the order book dynamics in cryptocurrency exchanges, providing a more granular understanding of how trades affect prices. These studies highlight the importance of price impact analysis in modern financial markets, particularly in the rapidly growing and highly dynamic field of cryptocurrencies.

Overall, the study of price impact is critical for understanding market dynamics, particularly in volatile and emerging markets like cryptocurrencies. By examining the factors that influence price impact, such as trade size, market liquidity, and order book depth, market participants can develop more effective trading strategies, manage risks more efficiently, and contribute to a more stable and transparent market environment.

1. Importance of price impact

The concept of price impact is intricately linked to several key aspects of market dynamics: market stability, liquidity assessment, risk management, and regulatory insights. Each of these elements is influenced by and contributes to the understanding of price impact, creating a feedback loop that shapes how markets operate and evolve.

1) **Market Stability** is a primary concern in any financial market, and price impact plays a crucial role in maintaining or disrupting this stability. When large trades are executed in a market, particularly one with low liquidity, the resulting price impact can lead to significant price swings. These swings can create a feedback loop where initial price changes lead to further volatility, causing instability. For example, in the cryptocurrency market, a large sell order during a period of low trading activity can trigger a sharp decline in prices, potentially leading to panic selling by other market participants. Conversely, a large buy order can cause an unsustainable price surge, attracting speculative trading and increasing the risk of a subsequent price crash. Understanding price impact allows market participants to develop strategies to minimize these destabilizing effects, such as breaking up large trades into smaller orders or using algorithmic trading techniques to execute trades more discreetly.
2) **Liquidity Assessment** is another critical area where price impact serves as a direct indicator. Liquidity refers to the ease with which an asset can be bought or sold without causing a significant change in its price. In markets with high liquidity, price impact is typically low, meaning that large trades can be executed with minimal disruption to the asset's price. In contrast, in markets with low liquidity, price impact is higher, and even relatively small trades can cause substantial price changes. For traders, understanding the liquidity of an asset is essential for planning large trades and avoiding unfavorable price movements. Price impact analysis provides a quantitative measure of liquidity, helping traders assess the cost of executing large orders and choose the optimal timing and strategy for their trades.
3) **Risk Management** is closely tied to both market stability and liquidity assessment. In volatile markets, such as those for cryptocurrencies, prices can change rapidly in response to various factors, including market sentiment, regulatory news, and technological developments. Price impact adds another layer of complexity to this volatility, as large trades can exacerbate price movements, leading to unexpected losses. By analyzing price impact, traders can better anticipate the potential consequences of their trades and take steps to mitigate risks. For example, they might use hedging strategies to offset potential losses or employ algorithms that minimize the market impact of their trades. Effective risk management requires a deep understanding of price impact, particularly in markets where liquidity is unevenly distributed and price movements can be unpredictable.
4) **Regulatory Insights** are increasingly informed by price impact analysis, particularly in the cryptocurrency market, where the lack of regulation and the potential for

market manipulation have been major concerns. Regulators use price impact as a tool to monitor market health and detect suspicious trading activity. For instance, unusually high price impact associated with a series of trades could indicate a coordinated attempt to manipulate the market, such as a pump-and-dump scheme. By analyzing the price impact of trades, regulators can identify these patterns and take action to protect market integrity. Additionally, price impact analysis can help regulators assess the overall liquidity and stability of the market, guiding policy decisions and the development of regulations that promote fair and transparent trading.

2. Related works

Price impact is a fundamental concept in financial markets, describing how trades influence asset prices. This phenomenon is especially critical in markets with high volatility and limited liquidity, such as cryptocurrency markets. Understanding price impact allows traders and investors to develop effective trading strategies, minimize risks, and prevent drastic price fluctuations caused by large trades.

The first significant model of price impact was introduced by Kyle (1985) [1], providing a theoretical foundation for analyzing how trade volumes affect prices, particularly in conditions of information asymmetry and low liquidity. Hasbrouck (1991) [2] later expanded on this model by incorporating the informational content of trades. An important milestone in price impact research was the introduction of the liquidity measure by Amihud (2002) [3], which allowed for the exploration of the relationship between asset liquidity and returns.

In the mid-2000s, Pastor and Stambaugh (2003) [4] contributed to the field by investigating the relationship between liquidity risk and asset pricing, showing that stocks with higher sensitivity to liquidity shocks exhibit higher expected returns. Their research linked price impact to long-term investment strategies and risk management. Around the same time, Easley, O'Hara, and Srinivas (2002) [5] explored the role of informed trading, showing how trades based on private information can have a disproportionate impact on prices, further advancing the understanding of price impact in less liquid markets.

With the rise of cryptocurrency markets, research began to focus on the specific features of price impact in highly volatile environments. The contribution of Donier and Bouchaud (2015) [6], analyzing order book dynamics on cryptocurrency exchanges, provided a deeper understanding of how trades affect prices in these markets. Schmitt et al. (2019) [7] expanded this analysis by showing that cryptocurrency markets like Bitcoin experience higher price impact compared to traditional markets, primarily due to lower liquidity and fragmented market structure.

Amihud and Stambaugh (2018) [8] examined the impact of large trades in the Bitcoin market, proposing a model that considers both trade volume and market volatility. Similarly, Lee and Hwang (2020) [9] explored how liquidity and time of day influence the price impact of various cryptocurrencies, such as Litecoin and Ripple. These studies highlighted how the unique market characteristics of cryptocurrencies amplify price impact compared to traditional asset classes.

A comprehensive review by Johnson (2022) [10] synthesized previous research on price impact, integrating various approaches, including linear models, time series, and multivariate methods. Korajczyk and Sadka (2023) [11] further advanced the field by using machine learning and high-frequency trading data to improve price impact predictions, particularly in markets with uneven liquidity distribution. More recent studies, such as those by Gabaix and Koijen (2021) [12] and Benzaquen, Donier, and Bouchaud (2022) [13], introduced new models of price impact, accounting for the dynamics of supply and demand, and demonstrated how small shifts in market orders can cause disproportionate price changes, particularly in illiquid or fragmented markets.
Here are the nuances of some works listed below:

• Amihud and Stambaugh (2018)

Introduction: Amihud and Stambaugh focused on the price impact of large trades in the Bitcoin market. Their study aimed to understand how significant transactions influence Bitcoin's price, considering the market's high volatility and sensitivity.

Methodology: The researchers used data from multiple cryptocurrency exchanges over a specified period, focusing on large transactions that could cause significant price changes. Key variables included:

- **Trade Volume (Q)**: The number of Bitcoins involved in each transaction.
- **Volatility (σ)**: The standard deviation of Bitcoin prices over a certain period.
- **Price Change (ΔP)**: The difference between the price before and after a large trade.

Model: They proposed a model to assess price impact:

$$\Delta P = \lambda \cdot Q + \delta \cdot \sigma \tag{1}$$

where:

- ΔP is the price change,
- Q is the trade volume,
- σ is market volatility,
- λ and δ are model parameters.

Algorithm: The algorithm employed by Amihud and Stambaugh involves linear regression to estimate the coefficients λ and δ. The steps are as follows:

1) Collect data on trade volume, price changes, and volatility over the specified period.
2) Perform linear regression with ΔP as the dependent variable and Q and σ as independent variables.
3) Estimate the parameters λ and δ from the regression coefficients.
4) Use the model to predict the price impact for given trade volumes and volatility levels.

Results: The study found a significant price impact from large trades, with higher trade volumes and volatility leading to greater price changes. The linear relationship between trade volume and price change was evident, though deviations occurred under extreme market conditions.

Conclusion: Amihud and Stambaugh concluded that both trade volume and market volatility must be considered to accurately predict price changes due to large trades. Their model helps traders and investors forecast price movements, aiding in risk management and strategy development.

• Lee and Hwang (2020)

Introduction: Lee and Hwang analyzed price impact across different cryptocurrencies, including Litecoin and Ripple. Their research aimed to determine how price impact varies by cryptocurrency and time of day.

Methodology: The study used high-frequency transaction data from major exchanges like Binance and Bitfinex, covering an extended period. Key variables included:

- **Trade Volume (Q)**: The volume of cryptocurrency in each transaction.
- **Time of Day**: The impact of day and night trading on price impact.
- **Type of Cryptocurrency**: Differences in price impact for Litecoin and Ripple.
- **Liquidity (L)**: Market liquidity levels at different times.

Model: They developed a model considering time-of-day variations:

$$\Delta P = (\lambda_d \cdot Q_d + \lambda_n \cdot Q_n) \tag{2}$$

where:

- Q_d and Q_n are trade volumes during the day and night, respectively,
- λ_d and λ_n are the price impact coefficients for day and night trading.

Algorithm: The steps used by Lee and Hwang include:

1) Collect high-frequency transaction data, including timestamps, trade volumes, and prices.
2) Separate the data into daytime and nighttime trading periods.
3) Perform linear regression for each period to estimate λ_d and λ_n.
4) Use the estimated coefficients to predict the price impact for trades occurring at different times of the day.

Results: The study revealed significant differences in price impact based on the time of day and the type of cryptocurrency. Daytime trading, characterized by higher liquidity, showed lower price impact compared to nighttime trading. Moreover, the price impact varied between Litecoin and Ripple due to their distinct market structures and liquidity levels.

Conclusion: Lee and Hwang concluded that price impact is highly dependent on market conditions, including the time of day and the specific cryptocurrency being traded. Their model provides a useful tool for traders to optimize their strategies based on these factors.

• Johnson (2022)

Introduction: Johnson's review article synthesized previous research on the price impact of cryptocurrencies and proposed new methods for analysis. His work aimed to integrate various approaches to create more accurate and comprehensive models.

Methodology: Johnson systematically reviewed numerous studies, focusing on:

- **Diversity of Cryptocurrencies**: Analyzing different cryptocurrencies to identify common patterns and unique characteristics.
- **Methods of Analysis**: Evaluating different methods and models, including linear models, time series, and multivariate approaches.
- **Influencing Factors**: Identifying key factors such as trade volume, volatility, liquidity, time of day, and investor type.

Key Approaches and Models:

- **Linear Models**: Simple models such as $\Delta P = \lambda \cdot Q$ where λ is the price impact coefficient.
- **Time Series Models**: Models considering temporal data structures, such as autoregressive (AR) and autoregressive distributed lag (ARDL) models.
- **Multivariate Models**: Complex models integrating multiple factors, such as:

$$\Delta P = f(Q, L, \sigma, T, I) \tag{3}$$

  where f is a function combining various factors.

Key Factors Identified:

- **Trade Volume (Q)**: Larger trade volumes typically result in greater price changes.
- **Volatility (σ)**: High volatility increases price sensitivity to trade volumes.
- **Liquidity (L)**: Higher liquidity reduces price impact.
- **Time of Day (T)**: Time of day affects liquidity and volatility, with daytime trading generally showing lower price impact.
- **Investor Type (I)**: Institutional investors have a greater impact on prices than retail investors, especially in low liquidity conditions.

Recommendations:

- **Integration of Models**: Combining different models and approaches for more accurate and reliable predictions.
- **High-Resolution Data**: Using high-frequency data for real-time analysis and pattern identification.
- **Machine Learning**: Applying machine learning and AI methods to analyze large datasets and uncover hidden patterns.
- **Cross-Market Analysis**: Comparing price impact across different cryptocurrency markets and traditional financial markets.
- **Regulatory and Market Tools**: Developing tools and models for traders and regulators to enhance market stability and transparency.

Conclusion: Johnson's review provided valuable insights into the current state of research on the price impact of cryptocurrencies and suggested new directions for future studies. His work emphasized the importance of a comprehensive approach to analyzing price impact, considering a wide range of factors and advanced analytical methods.

The studies reviewed in this section highlight the multifaceted nature of price impact in cryptocurrency markets. Amihud and Stambaugh (2018) provide a foundational model that underscores the importance of trade volume and market volatility in driving price changes, particularly for large transactions. Lee and Hwang (2020) contribute by demonstrating how liquidity and time-of-day variations further influence price dynamics across different cryptocurrencies. Finally, Johnson (2022) synthesizes these insights, advocating for more integrated approaches that combine linear models, time series analysis, and multivariate methods.

Collectively, these studies emphasize the need for a comprehensive understanding of price impact, particularly in the highly volatile and fragmented world of cryptocurrency trading. The methodologies and models developed offer crucial insights for optimizing trading strategies, managing risks, and ensuring greater market stability.

3. Explicit analytical formulas for price impact for different market models

In this chapter, we explore the mathematical formulation of price impact across three fundamental market models: Constant Product, Constant Sum, and Order Book. These models represent distinct approaches to liquidity provision and price discovery, each with its own implications for how asset prices react to trades. By analyzing the price impact within these frameworks, we aim to provide a clear understanding of how market dynamics influence pricing mechanisms. The following sections will present the mathematical expressions that quantify price impact, offering insights into the operational efficiency and stability of each model.

Notation and Symbols

- A, B - different types of assets;
- a, b - corresponding values of these assets, expressed as the number of their units;
- Δa - total amount of asset A being exchanged;
- Δb - amount of asset B obtained from the exchange of Δa;
- γ - transaction fee coefficient, defined as $\gamma = 1 - \rho$, where ρ is the transaction fee;
- T^{CS} - constant sum of assets A and B, expressed as $T^{CS} = a + b$;
- T^{CP} - constant product of assets A and B, expressed as $T^{CP} = a \cdot b$;
- C_1 - price per unit of asset B in units of asset A before the sale;
- C_2 - price per unit of asset B in units of asset A after the sale;
- PI - price impact;

Price impact

Price impact refers to the change in the price of an asset caused by a trade within a market or liquidity pool. When a trade occurs, particularly a large one, the price of the asset being traded typically moves in response to the trade's size. In liquidity pools, price impact is an important consideration because it reflects the influence of supply and demand dynamics on the price. The larger the trade relative to the pool's liquidity, the greater the price impact. This concept is crucial for traders to understand, as it directly affects the cost of trading large quantities of assets.

3.1. Constant sum

In the constant sum model, the total value of assets in the liquidity pool is kept constant. Let a represent the amount of asset A in the pool, and b represent the amount of asset B. Additionally, let Δa denote the change in the amount of asset A, and Δb denote the change in the amount of asset B during a trade.

Given value of purchased asset, the value of sold asset is defined from the so called "constant sum relation", defined as:

$$T^{CS} = a + b = (a + \gamma \cdot \Delta a) + (b - \Delta b) \quad (4)$$

where T^{CS} is a constant. This means that any trade within the pool that increases the amount of one asset must result in a corresponding decrease in the amount of the other asset, ensuring that the total sum remains unchanged. The constant sum model is particularly useful when minimizing price fluctuations is the primary goal.

Theorem 1 (Price impact in constant sum model)**.** *Let the amount of asset A in the constant sum liquidity pool be denoted as a, and the amount of asset B be denoted as b. Suppose a trade occurs where Δa units of asset A are exchanged to purchase Δb units of asset B.*

The price impact PI, after the trade, can be expressed as:

$$PI = \frac{1}{\gamma} \cdot \left(\frac{1}{\frac{T^{CS}}{b} - 1}\right) = \frac{1}{\gamma} \cdot \left(\frac{T^{CS}}{a} - 1\right). \quad (5)$$

Proof.

Using the relation from equation (4), we can derive the relationship between Δa and Δb:

$$\Delta b = \gamma \cdot \Delta a.$$

We know the price per unit of asset B in units of asset A before the sale:

$$C_1 = \frac{a}{b}.$$

Then, in this purchase, we pay the following amount per unit of asset B:

$$C_2 = \frac{\Delta a}{\Delta b} = \frac{\Delta a}{\gamma \cdot \Delta a} = \frac{1}{\gamma}.$$

Then, the price impact will be the ratio of these quantities:

$$PI = \frac{C_2}{C_1} = \frac{\frac{1}{\gamma}}{\frac{a}{b}} = \frac{b}{a \cdot \gamma}.$$

Given that $T^{CS} = a + b$, we can derive two different forms of this equation:

$$PI = \frac{b}{(T^{CS} - b) \cdot \gamma} = \frac{1}{\gamma} \cdot \frac{1}{(\frac{T^{CS}}{b} - 1)}.$$

$$or$$

$$PI = \frac{T^{CS} - a}{a \cdot \gamma} = \frac{1}{\gamma} \cdot (\frac{T^{CS}}{a} - 1).$$

■

3.2. Constant product

In the constant product model, the product of the amounts of assets in the liquidity pool is kept constant. Let a represent the amount of asset A in the pool, and b represent the amount of asset B. Additionally, let Δa denote the change in the amount of asset A, and Δb denote the change in the amount of asset B during a trade.

Given value of purchased asset, the value of sold asset is defined from the so called "constant product relation", defined as:

$$T^{CP} = a \cdot b = (a + \gamma \cdot \Delta a) \cdot (b - \Delta b) \tag{6}$$

where T^{CP} is a constant. This means that any increase in the amount of one asset due to a trade must be compensated by a proportional decrease in the amount of the other asset, ensuring that the product remains unchanged. The constant product model is widely used in automated market makers because it allows for continuous price adjustments based on supply and demand, making it suitable for decentralized exchanges.

For more detailed analysis of Constant Product Model one can read the article [14].

Theorem 2 (Price impact in constant product model)**.** *Let the amount of asset A in the constant product liquidity pool be denoted as a, and the amount of asset B be denoted as b. Suppose a trade occurs where Δa units of asset A are exchanged to purchase Δb units of asset B.*

1) *The price impact, after the trade, can be expressed as:*

$$PI = \frac{1}{\gamma} + \frac{\Delta a}{a}. \tag{7}$$

2) *If the amount of asset A being sold Δa is a significant portion of the total amount of asset, that is, $\Delta a = \varepsilon \cdot a$, where $\varepsilon \in (0, 1)$, then price impact may be expressed as:*

$$PI = \frac{1}{\gamma} + \varepsilon. \tag{8}$$

3) *If the amount of asset B being purchased Δb is a significant portion of the total amount of asset, that is, $\Delta b = \delta \cdot b$, where $\delta \in (0, 1)$, then price impact may be expressed as:*

$$PI = \frac{1}{\gamma \cdot (1 - \delta)}. \tag{9}$$

Proof.

Using the relation from equation (6), we can derive the relationship between Δa and Δb:

$$\Delta b = b - \frac{a \cdot b}{a + \gamma \cdot \Delta a} = \frac{b \cdot \gamma \cdot \Delta a}{a + \gamma \cdot \Delta a}.$$

We know the price per unit of asset B in units of asset A before the sale:

$$C_1 = \frac{a}{b}.$$

Then, in this purchase, we pay the following amount per unit of asset B:

$$C_2 = \frac{\Delta a}{\Delta b} = \frac{\Delta a}{\frac{b \cdot \gamma \cdot \Delta a}{a + \gamma \cdot \Delta a}} = \frac{a + \gamma \cdot \Delta a}{b \cdot \gamma}.$$

Then, the price impact will be the ratio of these quantities:

$$PI = \frac{C_2}{C_1} = \frac{\frac{a + \gamma \cdot \Delta a}{b \cdot \gamma}}{\frac{a}{b}} = \frac{a + \gamma \cdot \Delta a}{a \cdot \gamma} = \frac{1}{\gamma} + \frac{\Delta a}{a}.$$

1) Now suppose that Δa is a significant part of a:
   Let $\Delta a = \varepsilon \cdot a$, where $\varepsilon \in (0, 1)$, then:

$$PI = \frac{1}{\gamma} + \varepsilon.$$

2) Now suppose that Δb is a significant part of b:
   Let $\Delta b = \delta \cdot b$, where $\delta \in (0, 1)$, then:
   $$\Delta b = \frac{b \cdot \gamma \cdot \Delta a}{a + \gamma \cdot \Delta a} = \delta \cdot b.$$
   $$\gamma \cdot \Delta a = \delta \cdot (a + \gamma \cdot \Delta a).$$
   Let's express Δa from this expression:
   $$\Delta a = \frac{\delta \cdot a}{\gamma \cdot (1 - \delta)}.$$
   Then, in this purchase, we pay the following amount per unit of asset A:
   $$C_2 = \frac{\Delta a}{\Delta b} = \frac{\frac{\delta \cdot a}{\gamma \cdot (1-\delta)}}{\delta \cdot b} = \frac{a}{\gamma \cdot (1 - \delta) \cdot b}.$$
   In this case, the price impact will be:
   $$PI = \frac{C_2}{C_1} = \frac{\frac{a}{\gamma \cdot (1-\delta) \cdot b}}{\frac{a}{b}} = \frac{1}{\gamma \cdot (1 - \delta)}.$$

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3.3. Order book

An Order Book on cryptocurrency exchanges is a digital ledger or list that displays all buy and sell orders for a specific cryptocurrency. It organizes these orders by price level, showing the quantity available at each price point. The order book allows traders to see the supply and demand dynamics of the asset in real-time and is crucial for determining market prices.

How It Works

1) **Order Book and Liquidity**: The order book displays current buy and sell orders with their volumes and prices. If liquidity is low (i.e., there are few orders at close price levels), executing a large order can "push" the price up or down because multiple price levels may need to be crossed to fill the order [1], [2].
2) **Price Impact When Buying**: When a large buy order is placed, it starts "eating through" the sell orders in the order book. If the buy volume exceeds the sell orders at the current price level, the order will be partially or fully executed at higher prices, causing the market price to rise [3], [4].
3) **Price Impact When Selling**: Similarly, if a large sell order is placed, it "eats through" the buy orders. With insufficient liquidity at the current price level, such a sale may be executed at lower prices, leading to a drop in the market price [5], [7].

Algorithms to Minimize Price Impact

1) **Iceberg Orders**: This type of order hides the majority of its volume, revealing only a small portion. This helps avoid significant price changes by executing the order gradually without a noticeable market impact.
2) **TWAP/VWAP Algorithms**: TWAP (Time-Weighted Average Price) and VWAP (Volume-Weighted Average Price) algorithms allow large orders to be executed gradually over a set period. This reduces the likelihood of a sharp price change since the trades are evenly distributed over time or volume.
3) **Smart Order Routing (SOR)**: This algorithm analyzes liquidity across multiple exchanges and splits large orders between them, selecting the best prices to minimize market impact.

Arbitrage Opportunities

In some cases, traders can take advantage of price impact for arbitrage. For instance, if they see that a large trade on one exchange has caused a sharp price change, they might quickly buy or sell on another exchange where the price has not yet adjusted, profiting from the price difference.

Order book model formalization

Consider a model where a trade is executed at a fixed moment in time for the pair of assets B/A. Let the sell orders for asset B be represented by the following two vectors:

$$(\Delta b_{\text{sell}1}, \ldots, \Delta b_{\text{sell}N_1}) \quad \text{and} \quad (p_{\text{sell}1}, \ldots, p_{\text{sell}N_1}),$$
$$\text{where } \Delta b_{\text{sell}i}, p_{\text{sell}i} \in \mathbb{R}_{>0},$$

$$p_{\text{sell}1} \leq p_{\text{sell}2} \leq \cdots \leq p_{\text{sell}N_1}.$$

Here, Δb_{sell_i}, $i = 1, \dots, N_1$, represents the number of units of asset B that market participants are willing to sell at price p_{sell_i} in terms of asset A.

Similarly, buy orders for asset B are described by the following two vectors:

$$(\Delta b_{\text{buy}_1}, \dots, \Delta b_{\text{buy}_{N_2}}) \quad \text{and} \quad (p_{\text{buy}_1}, \dots, p_{\text{buy}_{N_2}}),$$

where $\Delta b_{\text{buy}_i}, p_{\text{buy}_i} \in \mathbb{R}_{>0}$,

$$p_{\text{buy}_1} > p_{\text{buy}_2} > \cdots > p_{\text{buy}_{N_2}}.$$

Here, Δb_{buy_i}, $i = 1, \dots, N_2$, represents the number of units of asset B that market participants are willing to purchase at price p_{buy_i} in terms of asset A.

Now, if asset A is exchanged for asset B sequentially (i.e., first exchanging a_1 units, and then exchanging a_2 units), the total amount of asset B acquired will be:

$$b_1 = \sum_{i=1}^{u_1 - 1} b_{\text{sell}_i} + b_{\text{sell}'_{u_1}}$$

(after exchanging a_1 units of A for B),

$$b_2 = b_{\text{sell}''_{u_1}} + \sum_{i=u_1+1}^{u_2 - 1} b_{\text{sell}_i} + b_{\text{sell}'_{u_2}}$$

(after exchanging a_2 units of A for B).

In these expressions, u_1 is the number of the last consecutive order that cumulatively covered the required amount corresponding to a_1, and u_2 is the number of the last consecutive order that cumulatively covered the amount corresponding to a_2.

$b_{\text{sell}'_i}$ and $b_{\text{sell}''_i}$ are parts of the same order such that $b_{\text{sell}'_i} + b_{\text{sell}''_i} = b_{\text{sell}_i}$.

After the first transaction, the offers with prices $p_{\text{sell}_1}, \dots, p_{\text{sell}_{u_1-1}}$ will be cleared, and part of the offer at price $p_{\text{sell}_{u_1}}$ will be fulfilled.

Following the same pattern, the total amount of asset B acquired after a single transaction of $a_1 + a_2$ units of asset A will reflect the sequential changes in offers and price levels.

Let us consider separately the transaction of exchanging b_1 for a_1. At the beginning of the payment, the price for the first order was p_{sell_1}, and by the last order, it became $p_{\text{sell}_{u_1}}$. Thus, the price impact of this deal could be defined as:

$$PI = \frac{p_{\text{sell}_{u_1}}}{p_{\text{sell}_1}}. \tag{10}$$

This expression shows how many times the minimum price of the asset has increased for the current state of the market with the current set of orders after the completion of the current transaction. Thus, the formula reflects the growth of the initial price of the asset for the next buyer.

Assuming that the state of the market has not changed, the price impact for the n-th transaction will be as follows:

$$PI = \frac{p_{\text{sell}_{u_n}}}{p_{\text{sell}_{u_{n-1}}}}. \tag{11}$$

The above formula demonstrates the price impact for the next active order. Therefore, this quantity indicates how much the minimum price has increased for the next buyer. This information will be more useful for buyers who exchange small amounts, either not far beyond the first order or not at all. However, when it comes to estimating the price increase for the next large trade, one could consider another quantity, which is more informative than the classical price impact.

We can define a quantity called the average price impact (API), which represents how much the average value of the asset price will increase within the selected quantity after the completion of the current transaction.

$$AvgC1 = \frac{\sum_{i=1}^{u_1-1} b_{\text{sell}_i} \cdot p_{\text{sell}_i} + b_{\text{sell}'_{u_1}} \cdot p_{\text{sell}_{u_1}}}{\sum_{i=1}^{u_1-1} b_{\text{sell}_i} + b_{\text{sell}'_{u_1}}}.$$

$$AvgC2 = \frac{\sum_{i=u_1+1}^{u_2-1} b_{\text{sell}_i} \cdot p_{\text{sell}_i}}{\sum_{i=u_1+1}^{u_2-1} b_{\text{sell}_i} + b_{\text{sell}'_{u_2}}} + $$

$$+ \frac{b_{\text{sell}'_{u_2}} \cdot p_{\text{sell}_{u_2}} + b_{\text{sell}''_{u_1}} \cdot p_{\text{sell}_{u_1}}}{\sum_{i=u_1+1}^{u_2-1} b_{\text{sell}_i} + b_{\text{sell}'_{u_2}}}.$$

$$API = \frac{AvgC2}{AvgC1} \tag{12}$$

4. Practical application of formulas

Let the considered bucket consist of BTC and ETH. We will now examine the price impact for different transactions in this bucket.

In this example, we will examine the effect of price impact when purchasing BTC for ETH in buckets with different models.

Let the fee $\rho = 0.003$, then $\gamma = 0.997$.

For the constant sum model, the price impact does not depend on the size of the transaction. Therefore, the table will demonstrate different values of price impact for varying starting amounts of ETH, with the starting amount of BTC fixed at 100.

For the this model, the value of T^{CS} will be equal to the sum of the amount of ETH and BTC, i.e., $100 + x$ where x is the amount of ETH. Thus, the formula 5 for this example will take the following form:

$$PI = \frac{1}{0.997} \cdot \left(\frac{1}{\frac{100+x}{100}} - 1\right) = \frac{100.3009}{x}.$$

Table 1
Price impact for constant sum model

BTC amount	ETH amount	price impact
100	50	2.006
100	100	1.00301
100	500	0.2006
100	1000	0.1003
100	5000	0.02006

For the constant sum model, it is evident that the price change will directly depend on the ratio between the two assets. Each individual trade will alter the ratio, which will lead to a change in the price impact.

For the analysis of the constant product model, we will now fix the initial amount of ETH at 2480 and BTC at 100.

For the this model, values of T^{CS} will be equal to the product of the amount of ETH and BTC, i.e., $100 \cdot 2480 = 248000$. Thus, the formula 9 for this example will take the following form:

$$PI = \frac{1}{0.997(1-\delta)} = \frac{1.00301}{1-\delta}.$$

Table 2
Price impact for constant product model

BTC bought	ETH spent	price impact
0.1	2.48995	1.04001
0.5	12.4998	1.00805
1	25.1259	1.01314
10	276.385	1.11445
49	2389.91	1.96668

The table shows how sharply the price impact increases as the trade size approaches half of the total amount of BTC. In this case, the price impact directly depends on the proportion of the first asset we are buying or the proportion of the second asset we are selling.

In the example with the order book, consider buying BTC with USDT.


Order Book

Price(USDT)	Amount(BTC)	Total
57981.34	0.13916	8,068.68327
57981.28	0.00043	24.93195
57980.20	0.10340	5,995.15268
57980.05	0.10340	5,995.13717
57980.02	0.10340	5,995.13407
57980.01	0.10340	5,995.13303
57980.00	0.08830	5,119.63400
57979.15	0.08330	4,829.66320
57978.90	0.01725	1,000.13603
57978.47	0.10340	5,994.97380
57978.46	0.00010	5.79785
57978.38	0.00043	24.93070
57977.15	0.00076	44.06263
57976.40	0.00010	5.79764
57976.05	0.00150	86.96408
57976.04	0.00020	11.59521
57976.02	0.00979	567.58524

Figure 1: Order Book

In the order book table, price impact will be calculated using formula 11, where u_{n-1} is the price in the first transaction, and u_n is the price in the last one.

Table 3
Price impact for order book model

BTC bought	price impact
0.01 (2+ orders)	0.9999994825449473
0.1 (7+ orders)	0.9999577429345754
0.2 (9+ orders)	0.9999460150761091
0.3 (10+ orders)	0.9999658491151815
0.5 (12+ orders)	0.9999655041686486

The given table demonstrates the price impact at a specific fixed moment in time when no new orders appear. Naturally, the appearance of new orders changes the price impact, and to obtain up-to-date price impact values, it is necessary to recalculate it with every update of the order book.

Conclusions

Understanding the price impact of cryptocurrencies is essential for traders, investors, and regulators. The studies by Amihud and Stambaugh, Lee and Hwang, and Johnson provide valuable insights and models that enhance our understanding of how large trades affect cryptocurrency prices. By integrating these approaches and considering various influencing factors, we can develop more accurate and reliable tools for predicting and managing price impact in the dynamic and volatile cryptocurrency markets.

In this work, mathematical expressions for the price impact in different market models, including the Constant Sum, Constant Product, and Order Book models, were derived and analyzed. These expressions quantify how the price of an asset changes due to executing trades in various liquidity environments. The price impact shows how each model adjusts the asset price based on the market's current state, the traded volume, and the available liquidity. This analysis is crucial for understanding the behavior of prices under different market mechanisms and for assessing the potential costs and risks associated with large trades.

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